Receiver Sensitivity and Equivalent Noise Bandwidth

Parent Category: 2014 HFE

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Introduction

Receivers often contain narrow bandpass hardware filters as well as narrow lowpass filters implemented in digital signal processing (DSP). The equivalent noise bandwidth (ENBW) is a way to understand the noise floor that is present in these filters. To predict the sensitivity of a receiver design it is critical to understand noise including ENBW. This paper will cover each of the building block characteristics used to calculate receiver sensitivity and then put them together to make the calculation.

Receiver Sensitivity

Receiver sensitivity is a measure of the ability of a receiver to demodulate and get information from a weak signal. We quantify sensitivity as the lowest signal power level from which we can get useful information. In an Analog FM system the standard figure of merit for usable information is SINAD, a ratio of demodulated audio signal to noise. In digital systems receive signal quality is measured by calculating the ratio of bits received that are wrong to the total number of bits received. This is called Bit Error Rate (BER). Most Land Mobile radio systems use one of these figures of merit to quantify sensitivity. To measure sensitivity, we apply a desired signal and reduce the signal power until the quality threshold is met.

SINAD

SINAD is a term used for the Signal to Noise and Distortion ratio and is a type of audio signal to noise ratio. In an analog FM system, demodulated audio signal to noise ratio is an indication of RF signal quality. In order to measure the audio signal to noise ratio, typically test equipment measures total audio power (Signal plus Noise plus Distortion) and then notch filters the audio signal tone (typically 1 kHz) and measures the audio power again (Noise plus Distortion) and takes the ratio in decibels.

$$SINAD(dB) = 10 * log10\left(\frac{Signal+Noise+Distortion}{Noise+Distortion}\right)$$

Land Mobile radio industry standards typically use 12 dB SINAD for the measurement of reference sensitivity.

BER

Bit Error Rate is a measure of signal to noise ratio in a digital modulation system. In order to calculate the BER, a known repeating pattern must be transmitted to the radio. The receiver must demodulate the data and compare it to the known data pattern and determine the number of bits that are errors. The BER is then the ratio of bits in error to total bits received. The industry standard for Land Mobile radio is typically 5% BER for reference sensitivity.

Calculate Receiver Sensitivity

The sensitivity of a receiver can be calculated if one knows the following performance parameters: the noise figure (NF), the ENBW, and the carrier to noise ratio (C/N) required to achieve the desired quality signal.

The sensitivity is as follows:

$\text{Sensitivity}=10\times\log10(kTB)+NF+C/N$

This equation defines the signal power in dB·Watts that is present at the demodulator for a desired carrier to noise ratio. Let’s explain each of the terms in this equation.

What is “kTB”?

The total thermal noise power (kTB) is a function of three quantities, 1) Boltzmann’s constant “k” in Joules/°K, 2) temperature in °Kelvin, and 3) the overall bandwidth of the channel selective filtering in the receiver. This is referred to as “Thermal Noise” because of the dependency on temperature.

Thermal Noise floor=k(Joules⁄°K)×T(°K)×B(Hz)

The resulting noise is in Joules/Second or Watts. To convert the noise power to dB·Watts, use 10 times the log of the noise power in watts. If we look at the normalized (B = 1 Hz bandwidth) noise floor equation, we have:

Noisefloor=10×log10(k×T×B)

=10×log10(1.38× 〖10-23×290°×1 Hz)

= -203.9 dBW⁄Hz

Next, to convert from dBWatts to dBmilliwatts (dBm) increase this value by 30 dB:

–203.9 dBW⁄Hz+30 dB= -173.9 dBm⁄Hz

This is the amount of noise power in a 1 Hz bandwidth.

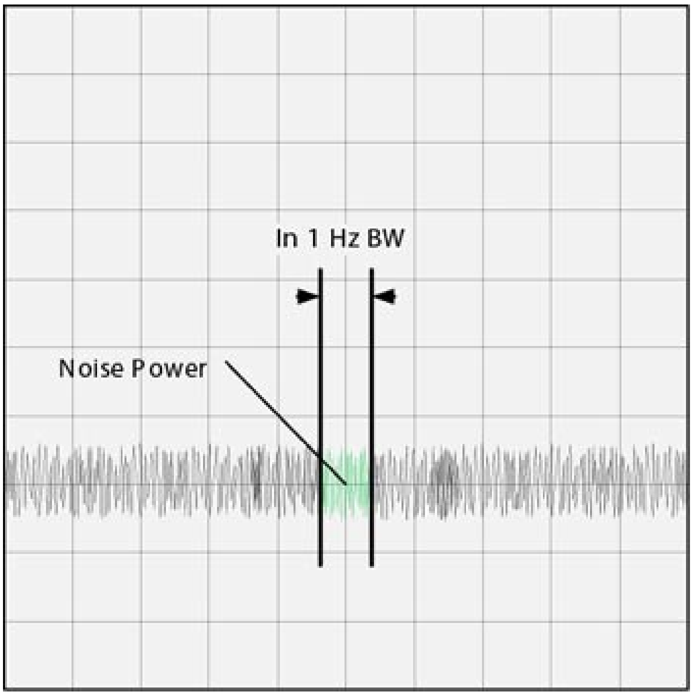


Figure 1 • Noise power in a 1 Hz bandwidth.

What is Noise Figure?

The Noise figure is the amount of noise power added by the electronic circuitry in the receiver to the thermal noise power from the input of the receiver. The thermal noise at the input to the receiver passes through to the demodulator. This noise is present in the receive channel and cannot be removed. The noise figure of circuits in the receiver such as amplifiers and mixers, adds additional noise to the receive channel. This raises the noise floor at the demodulator.

What is Carrier to Noise Ratio (C/N)?

In order to achieve the desired quality of demodulated signal, the signal power must be higher than the noise floor. The required ratio of signal power to noise floor is known for certain types of modulation. For an analog FM land mobile radio system using 25 kHz channels, the receiver must have approximately 4 dB more signal power than noise power. This represents a carrier to noise ratio 4 dB.

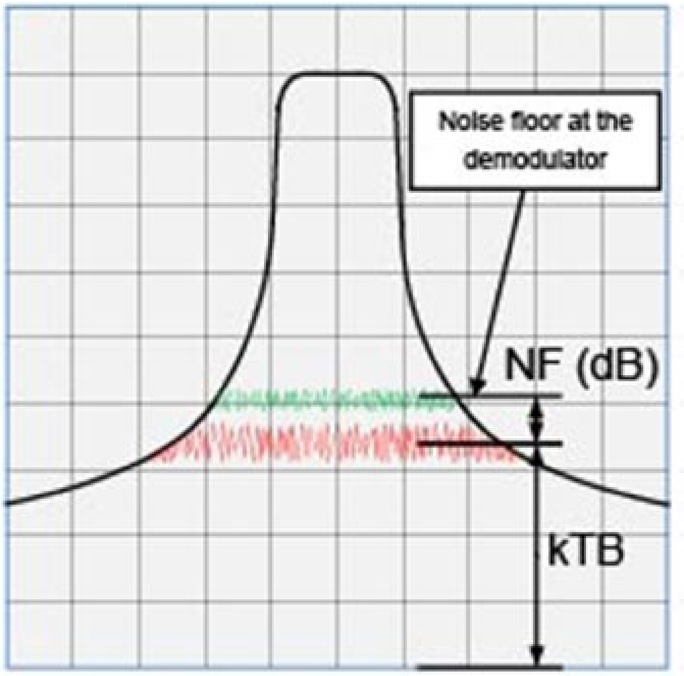


Figure 2 • Noise Figure added to thermal noise (kTB).

Bit Error Rate (BER) is the sensitivity benchmark for digital modulation systems. Eb/No is the ratio of the Energy per bit (Eb) to the noise spectral density (No - the noise power present in 1 Hz). The carrier to noise ratio required for a certain BER is a function of the Eb/No of the signal. This is a digital system representation of signal to noise ratio. Each digital modulation type has a Eb/No curve (Eb/No vs. BER). In order to determine sensitivity, use the appropriate curve and find desired bit error rate to determine the necessary Eb/No. Then calculate the Carrier to noise ratio by the following relationship:

$$Carrier/Noise_{dB} = 10\times log10\left(\frac{E_b}{N_o}\right) + 10\times log10\left(\frac{F_b}{B}\right)$$

Where Fb is the bit rate and B is the receiver equivalent noise bandwidth.

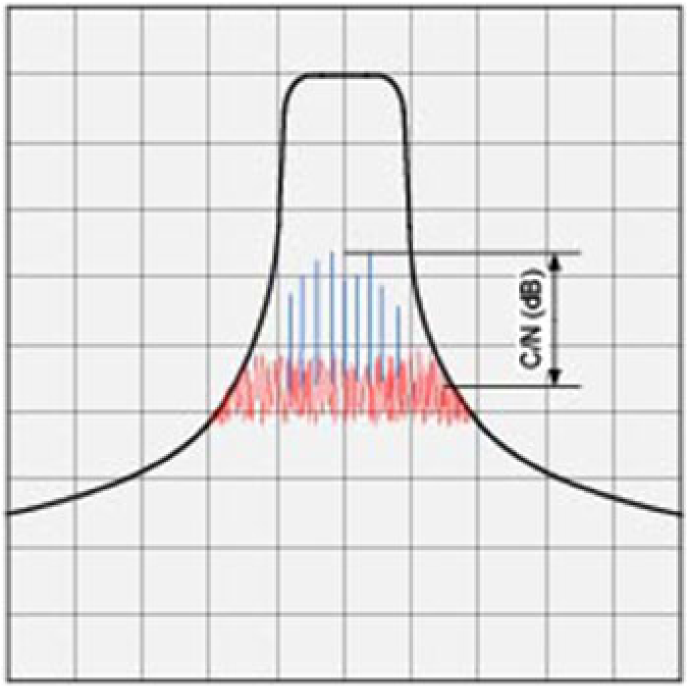


Figure 3 • Carrier to Noise ratio.

What is Equivalent Noise Bandwidth?

A filter’s equivalent noise bandwidth (ENBW) is defined as the bandwidth of a perfect rectangular filter that passes the same amount of power as the cumulative bandwidth of the channel selective filters in the receiver.

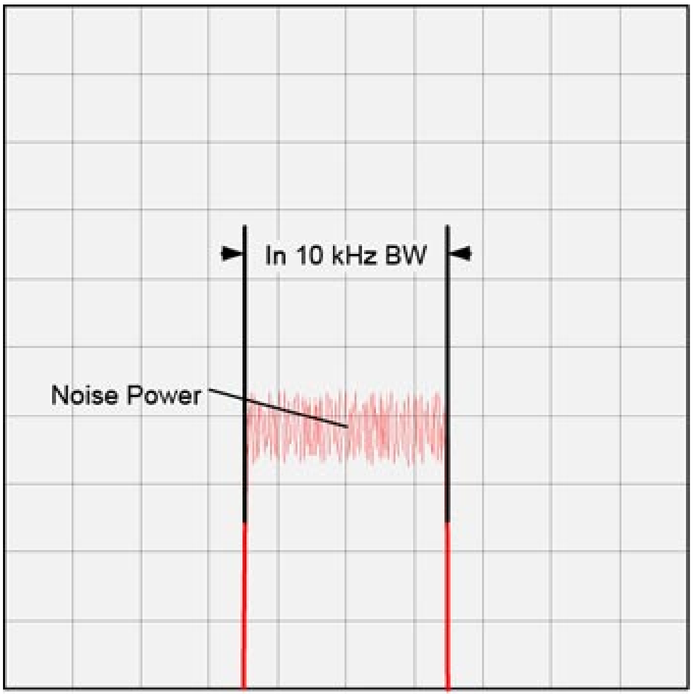


Figure 4 • Noise Power in the IF Bandwidth.

At this point we would like to know the noise floor in our receiver, i.e. the noise power in the receiver intermediate frequency (IF) filter bandwidth that comes from kTB. Since the units of kTB are Watts/ Hz, calculate the noise floor in the channel bandwidth by multiplying the noise power in a 1 Hz bandwidth by the overall equivalent noise bandwidth in Hz. For a receiver with a 10 kHz ENBW, we calculate the noise floor in dB milliwatts (dBm) as follows:

Noisefloor=10×log10(1.38× 〖10-23×290˚×1 Hz×10000)+30 = –134.0 dBm

Next we see how the bandwidth of a perfect rectangular filter compares to the actual filter response of the channel selective filters in the receiver.

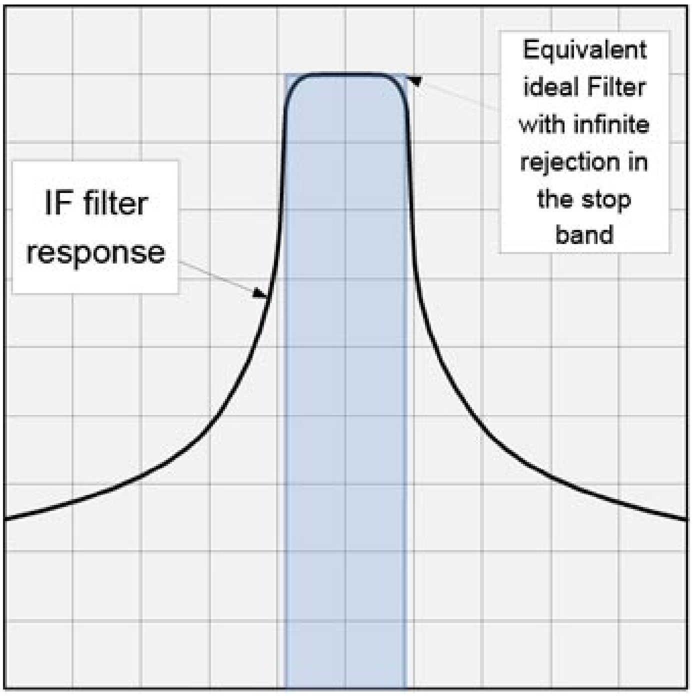


Figure 5 • Perfect filter that passes same power.

We use the bandwidth of the equivalent ideal rectangular filter (ENBW) to calculate the thermal noise floor. We may specify the equivalent noise bandwidth for design purposes but in practice, it is the composite bandwidth of all of the filters in front of the demodulator. The power that a filter can pass is a function of the area under the filter curve. The filter plot is in dB.

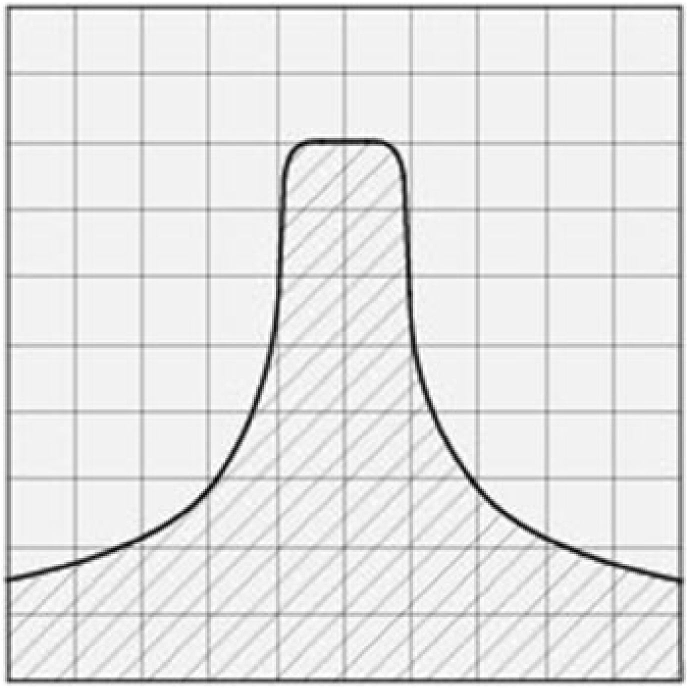


Figure 6 • Area under filter response.

Calculating ENBW from Measured Data

Ideally, we calculate the 2-sided ENBW by integrating the normalized filter power frequency response curve from -infinity to infinity (–fs/2 to fs/2 for a digital filter sampled at a rate of fs). For practical purposes the -60 dB BW values of the normalized filter response can be used as the limits of integration. Since we are looking for bandwidth in Hertz, we do not need to know the absolute power under the curve. The integration must be done in linear terms of watts or milliwatts not dB. Scattering parameters or S-parameters are a measurement of how radio frequency (RF) voltage propagates through an RF network. S-parameters of an RF filter can easily be measured using a network analyzer. We then calculate the bandwidth using the measured S-parameter data. Since S-parameters are voltage related measurements, we can convert them to a power quantity by the relationship:

$$Power = \frac{Voltage}{Resistance}^{2}$$

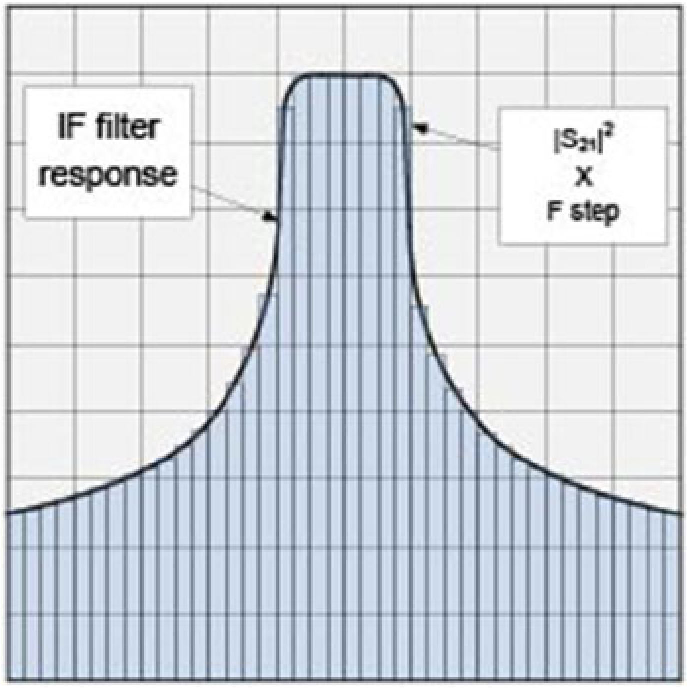


Figure 7 • Integration by summation of rectangles.

We can use the magnitude of the through response (S21) as the voltage term and normalize the impedance to 1 Ohm to substitute into the Power equation. This will give us a linear power term. For our case we choose S parameters where the magnitude is in dB. The power must be converted to a linear term for our calculations.

$$|S_{21}|linear = 10^{\frac{S_{21}(dB)}{20}}$$

For the overall ENBW, we want a rectangle where the height of the rectangle is equal to the maximum power (|S21|2). The area of the rectangular filter is equal to the area under the filter curve. Calculate the area of the under the IF filter response curve by using numerical integration by uniform rectangles.

Find the area of each uniform rectangle by multiplying |S21|2 (power) by the frequency step size used in the S parameter data.

$$Area\ under\ the\ filter\ curve = \sum(|S_{21}|^2 \times Freq\ step)$$

For a simple rectangle:

$$Width = \frac{area}{height}$$

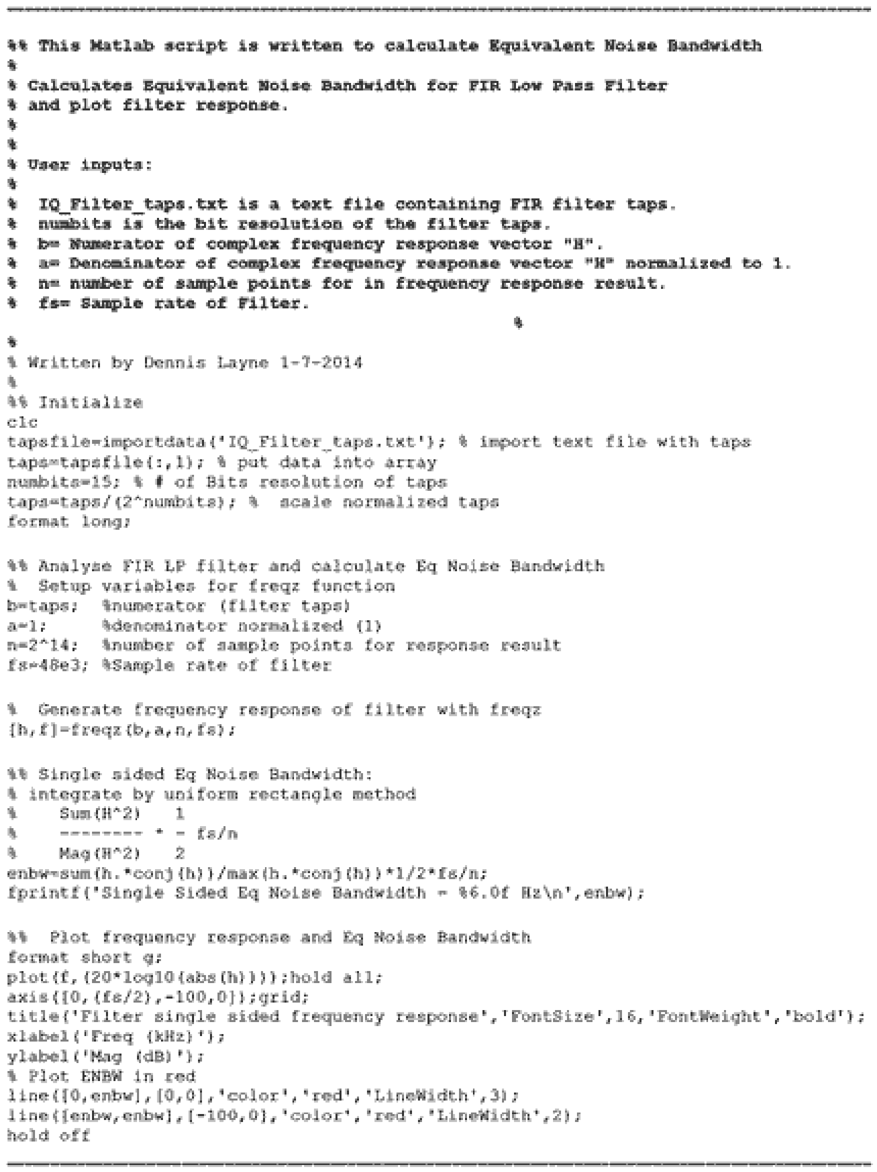
Therefore, the ENBW is:

Therefore, the ENBW is:

$$Bandwidth\ (Hertz) = \frac{\sum(|S_{21}|^2 \times Freq\ step)}{maximum\ |S_{21}|^2}$$

Since the S21 terms are unitless we now have a result that is frequency in Hertz. The result is the bandwidth of a rectangular filter with infinite stopband rejection that passes the same amount of power as the filter that we measured (S parameters). It is this bandwidth that we will use in our calculation of the receiver noise floor (kTB).

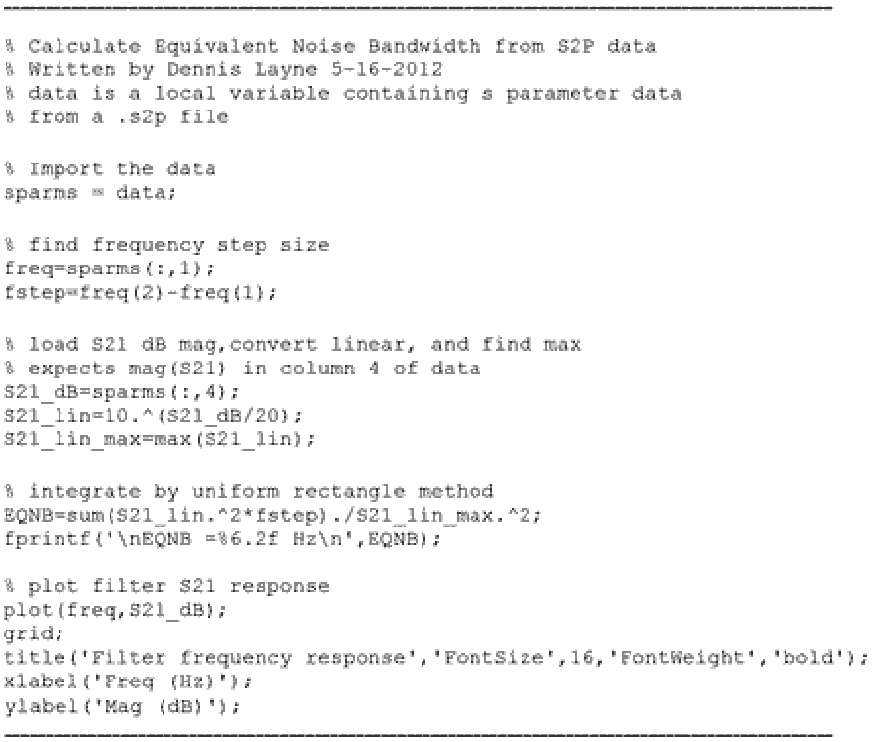
We may use software to calculate a filter’s ENBW. Here is a Matlab script for calculating ENBW from an
“.s2p” s-parameter data file:



```
%% This Matlab script is written to calculate Equivalent Noise Bandwidth
%
% Calculates Equivalent Noise Bandwidth for FIR Low Pass Filter
% and plot filter response.
%
%
% User inputs:
%
%  IQ_Filter_taps.txt is a text file containing FIR filter taps.
%  numbits is the bit resolution of the filter taps.
%  b= Numerator of complex frequency response vector "H".
%  a= Denominator of complex frequency response vector "H" normalized to 1.
%  n= number of sample points for in frequency response result.
%  fs= Sample rate of Filter.
                                                %
%
% Written by Dennis Layne 1-7-2014
%
%% Initialize
clc
tapsfile=importdata('IQ_Filter_taps.txt'); % import text file with taps
taps=tapsfile(:,1); % put data into array
numbits=15; % # of Bits resolution of taps
taps=taps/(2^numbits); %  scale normalized taps
format long;

%% Analyse FIR LP filter and calculate Eq Noise Bandwidth
%  Setup variables for freqz function
b=taps;  %numerator (filter taps)
a=1;     %denominator normalized (1)
n=2^14;  %number of sample points for response result
fs=48e3; %Sample rate of filter

%  Generate frequency response of filter with freqz
[h,f]=freqz(b,a,n,fs);

%% Single sided Eq Noise Bandwidth:
% integrate by uniform rectangle method
%    Sum(H^2)    1
%    -------- * - fs/n
%    Mag(H^2)    2
enbw=sum(h.*conj(h))/max(h.*conj(h))*1/2*fs/n;
fprintf('Single Sided Eq Noise Bandwidth = %6.0f Hz\n',enbw);

%%  Plot frequency response and Eq Noise Bandwidth
format short g;
plot(f,(20*log10(abs(h))));hold all;
axis([0,(fs/2),-100,0]);grid;
title('Filter single sided frequency response','FontSize',16,'FontWeight','bold');
xlabel('Freq (kHz)');
ylabel('Mag (dB)');
% Plot ENBW in red
line([0,enbw],[0,0],'color','red','LineWidth',3);
line([enbw,enbw],[-100,0],'color','red','LineWidth',2);
hold off
```

Figure 8 • Matlab code to calculate ENBW of a filter using s parameters.

Sometimes a digital filter implemented as a lowpass filter in DSP is the narrowest filter in a given system. Here is a Matlab script that will calculate the ENBW of a digital lowpass filter given a text file that contains the taps for the filter:



```
% Calculate Equivalent Noise Bandwidth from S2P data
% Written by Dennis Layne 5-16-2012
% data is a local variable containing s parameter data
% from a .s2p file

% Import the data
sparms = data;

% find frequency step size
freq=sparms(:,1);
fstep=freq(2)-freq(1);

% load S21 dB mag,convert linear, and find max
% expects mag(S21) in column 4 of data
S21_dB=sparms(:,4);
S21_lin=10.^(S21_dB/20);
S21_lin_max=max(S21_lin);

% integrate by uniform rectangle method
EQNB=sum(S21_lin.^2*fstep)./S21_lin_max.^2;
fprintf('\nEQNB =%6.2f Hz\n',EQNB);

% plot filter S21 response
plot(freq,S21_dB);
grid;
title('Filter frequency response','FontSize',16,'FontWeight','bold');
xlabel('Freq (Hz)');
ylabel('Mag (dB)');
```

Figure 9 • Matlab code to calculate ENBW of a filter using FIR filter taps.

Now We Can Calculate Receiver Sensitivity

We have quantities for each of the elements of the equation for receiver sensitivity. To calculate receiver sensitivity, we add the overall noise figure of the receiver to the noise floor. This quantifies the noise floor at the input to the demodulator. The signal must be higher than the noise floor by the carrier to noise ratio required for a desired signal quality. When these things are added, the result is the power level required to meet the figure of merit for usable information referred to as Reference Sensitivity.

$\text{Sensitivity}=10\times\log10(kTB)+30+NF+C/N$

We can calculate the sensitivity of a receiver with a 5 dB noise figure (NF) for analog FM in a 25 kHz channel, using the noise floor in a 10 kHz ENBW receiver that we calculated earlier. The carrier to noise ratio required for 12 dB SINAD in an analog FM receiver with a 10 kHz ENBW is approximately 4 dB. Substitute the values into

$\text{Sensitivity}=10\times\log10(kTB)+30+NF+C/N$

Equation 10 as follows:

$\text{Sensitivity}=10\times\log10(kT\times10000)+30\text{ dB}+5\text{ dB}+4\text{ dB}$

$= -134.0\text{ dBm}+5\text{ dB}+4\text{ dB}$

$= -125.0\text{ dBm}$

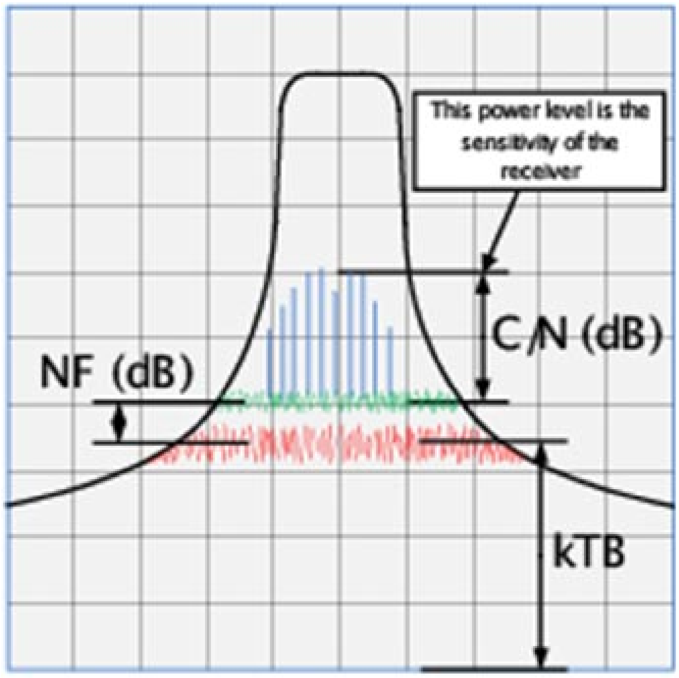


Figure 10 • Thermal Noise (kTB) plus NF plus C/N.

Conclusion

The sensitivity of a receiver is a function of band limited thermal noise, receiver noise figure, and the required carrier to noise ratio for a particular modulation. The equivalent noise bandwidth refers to the amount that the noise is band limited. It is possible to estimate the ENBW, but it can be calculated from measured data or DSP filter taps. Using these characteristics, we can accurately calculate sensitivity of a receiver.

About the Author:

Dennis Layne is a Senior Principal RF Engineer at Harris Corporation. Dennis has a Bachelor's of Science in Physics from Lynchburg College in Virginia and has 18 years experience designing receivers for Land Mobile Radio.